Quantum Computer with Fixed Interaction is Universal

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Abstract

It is proved that a quantum computer with fixed and permanent interaction of diagonal type between qubits proposed in the work quant-ph/0201132 is universal. Such computer is controlled only by one-qubit quick transformations, and this makes it feasible.

1 Introduction and background

A model of quantum computer with fixed and permanent interaction between qubits was proposed in the paper [1] where it was shown how to implement QFT and simulation of wave functions dynamics by such computer. In this paper we prove that such a model is universal that is every quantum algorithm can be implemented in the framework of this model with only linear slowdown for long-distance interaction and with the slowdown as multiplication by a size of memory for short distance interaction. Here we have to suppress undesirable interactions like it is done in the work [2]. But now we shall use the method of random strings consisting of NOT operations proposed in [1] which uses a diagonal form of interactions. Surprisingly, that our method of suppressing undesirable interactions does not depend on individual features of qubits.

A traditional way for implementation of quantum algorithms requires a control on two qubits level that is an ability to "switch on" and to "switch off" interaction between qubits. Whereas is widely known that two qubits transformations is a stumbling block in quantum computing in view of technological difficulties. The most natural way is to use a fixed and permanent interaction between qubits and control a process of computation by only one-qubit transformations. This way gives a universal quantum computer if our fixed interaction has a

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diagonal type. Note that it is no matter how a fixed interaction decreases depending on the distance between qubits, for example it may be nonzero only for the neighboring qubits, etc.

A permanent interaction between qubits in our computer depends only on their spatial disposition which is fixed. The only condition we impose to the interaction is that it must be diagonal. Thus if j and k denote identification numbers of two qubits then the Hamiltonian of their interaction will have the form

$$\text{A) } H_{j,k} = \begin{pmatrix} E_1^{j,k} & 0 & 0 & 0 \\ 0 & E_2^{j,k} & 0 & 0 \\ 0 & 0 & E_3^{j,k} & 0 \\ 0 & 0 & 0 & E_4^{j,k} \end{pmatrix}, \qquad \text{B) } H_{j,k} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & E_{j,k} \end{pmatrix}, \quad E_{j,k} > 0. \tag{1}$$

At first note that any interaction of the general form (1, A) may be reduced to the form (1, B) by adding appropriate one-qubit Hamiltonians $H'_{j,k}$ which matrices have the forms

$$\begin{pmatrix} a & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & b \end{pmatrix}, \qquad \begin{pmatrix} \alpha & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 \\ 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & \beta \end{pmatrix}.$$

This addition reduces Hamiltonian of the form (1, A) to (1, B) and it can be alternatively fulfilled by one-qubit quick transformations because all these diagonal matrices commute.

Note that the different pairs of qubits may interact variously, they may be disposed with the different intervals and be not placed along one line, etc.

2 Suppression of undesirable interactions by one qubit operations

To prove a universality of computational model we must show how one can fulfil an arbitrary two qubits operation. Given a unitary transform induced by Hamiltonian (1, B) in time frame 1: $U_{j,k} = \exp(-iH_{j,k})$ (Plank constant equals 1). In fact it would suffice to fulfil this transform on two qubits: jth and kth preserving all others untouched. Just this last condition is difficult to guarantee for permanent interaction. If we can do it when at first we can fulfil an arbitrary two qubits operation with every separated pair of qubits. Then for a long distance interaction we shall have at most a linear slowdown, for a short distance interaction we shall need to perform SWAP operations to bring a required pair of qubits together and thus obtain time factor equal to a size of memory.

Now show how to implement $U_{j,k}$. If we simply wait for a time 1, when we obtain a transformation $U_{j,k} \bigotimes U' \bigotimes \ldots \bigotimes U''$ where all U' have the form $U_{j',k'}$ where $\{j', k'\} \neq \{j, k\}$. We should get rid of these interactions. Declare jth and kth qubits separated.

We shall apply one qubit gate NOT several times to all qubits but separated ones to suppress all two qubits interactions excluding interaction between separated qubits. For each not separated qubit number p consider the Poisson random process $\mathcal{A}_p$ generating time

instants $0 < t_1^p < t_2^p < \ldots < t_{m_p}^p < 1$ with some fixed density $\lambda \gg 1$. Let all $\mathcal{A}_p$ are independent. Now fulfil transformations NOT on each pth qubit in instants t_m^p sequentially. In instant 1 fulfil NOT on pth qubit if and only if m_p is odd. Then after this procedure each qth qubit restores its initial value a_q. Count the phase shift generated by this procedure. Interaction between separated qubits remains unchanged. Fix some not separated qubit number p and count its deposit to phase. It consists of two summands: the first comes from interaction with separated and the second - from interaction with not separated qubits. Count them sequentially.

1. In view of big density λ of Poisson process $\mathcal{A}_p$ about half of time our pth qubit will be in state a_p and the rest half - in $1 - a_p$. Its interaction with a separated qubit, say jth brings the deposit $\frac{1}{2}E_{p,j}a'_pa'_j + \frac{1}{2}E_{p,j}(1 - a'_p)a'_j$ that is $\frac{1}{2}E_{p,j}a'_j$.

2. Consider a different not separated qubit number $q \neq p$. In view of independence of time instants when NOTs are fulfiled on pth and qth qubits and big density λ these qubits will be in each of states $(a_p,\ a_q)$, $(a_p,\ 1 - a_q)$, $(1 - a_p,\ a_q)$, $(1 - a_p,\ 1 - a_q)$ approximately a quarter of time. The resulting deposit will be $\frac{1}{4}E_{p,q}[a_pa_q + a_p(1 - a_q) + (1 - a_p)a_q + (1 - a_p)(1 - a_q)]$ $= \frac{1}{4}E_{p,q}$.

A total phase shift issued from the presence of not separated qubits in our procedure now is obtained by summing values from items 1 and 2 for all $p \notin \{j, k\}$. It is

$$\frac{1}{2}[\sum_{p\notin\{j,k\}} E_{p,j}a_j + \sum_{p\notin\{j,k\}} E_{p,k}a_k] + \frac{1}{4}\sum_{p,q\notin\{j,k\}} E_{p,q}.$$

This shift can be compensated by one-qubit operations because the first two summands depend linearly on the qubits values and the second does not depend on qubits values at all. Thus we obtain a gate with permanent two qubits interaction and one-qubit operations fulfiling phase shift to $d_{j,k}a_ja_k$ that is required. If we take time frame Δt instead of unit time in this procedure we obtain the phase shift to $\Delta t\ E_{j,k}a_ja_k$.

Thus we can implement $U_{j,k}$ for every separated pair of qubits.

3 Implementation of CNOT by fixed interaction

Now show in details how to implement CNOT gate with a given pair of qubits. Let j, k be fixed and omit these indexes. Denote $\Delta E = E_1 - E_2 - E_3 + E_4$. If $\frac{\Delta E}{\pi} \notin Q$ ($\frac{\Delta E}{\pi}$ is not rational number), then (as some physical parameters of our system, infulencing phases, for example, cycle period can be slighltly varied to avoid rationality of this parameter, the opposite case (rationality) can be ignored without lack of generality) we can effectively implement common two-qubit gate controlled-NOT (CNOT)

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

over our pair of neighbour qubits by using sequence of gates only from given set of arbitrary one-qubit rotations and fixed diagonal two-qubit gate E

$$E = \begin{pmatrix} \exp(iE_1) & 0 & 0 & 0 \\ 0 & \exp(iE_2) & 0 & 0 \\ 0 & 0 & \exp(iE_3) & 0 \\ 0 & 0 & 0 & \exp(iE_4) \end{pmatrix}$$

by the following way.

I. Denote gate implementing by sequential implementation of first qubit phase rotation A

$$A = \begin{pmatrix} 1 & 0 \\ 0 & \exp(i(E_1 - E_3)) \end{pmatrix},$$

second qubit phase rotation B

$$B = \begin{pmatrix} \exp(-iE_1) & 0 \\ 0 & \exp(-iE_2) \end{pmatrix},$$

and gate E as U

$$U = E(A \bigotimes B) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \exp(i\Delta E) \end{pmatrix}.$$

II. By using irrationality of $\frac{\Delta E}{\pi}$ it can be shown that

$$\forall \varepsilon > 0 \exists m \in N \exists n \in N : |\Delta E n - \pi(2m+1)| < \varepsilon,$$

i.e. for any desired accuracy ε there exists $n = n(\varepsilon)$ so that U^n will approach Π gate

$$\Pi = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

with given accuracy.

III. By using relation

$$(I \bigotimes H)\Pi(I \bigotimes H) = CNOT,$$

where I is identity matrix

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

and H is Hadamard gate

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

or, in matrix form,

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1&1&0&0\\1&-1&0&0\\0&0&1&1\\0&0&1&-1\end{pmatrix}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-1\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1&1&0&0\\1&-1&0&0\\0&0&1&1\\0&0&1&-1\end{pmatrix}=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}$$

we see that controlled-NOT is finally obtained by the sequence

$$(I\bigotimes H)\left(E\,(A\bigotimes B)\right)^{n}(I\bigotimes H)$$

of one-qubit rotations and gate E.

4 Conclusion

It is established that a quantum computer controlled by quick one-qubit transformations and with fixed permanent interaction of diagonal form between qubits is universal. It means that this type of quantum computer can implement all possible quantum algorithms by switching on and off only one-qubit gates.

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References

[1] Y. Ozhigov, Implementation of Quantum Fourier Transform and Simulation of Wavefunction by Fixed Interaction *LANL e-print quant-ph/0201132*

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